Confidence Intervals for the Mean
Sections 16.3, 16.4

Lecture 30

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Outline

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The Level of Confidence

- We have used 2 standard deviation to obtain a 95% confidence interval.
- Two problems with that:

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- Two problems with that:
  1. “2” is an approximation.
  2. What if we want 99% confidence or some other level of confidence?

The Level of Confidence

- To find the 95% confidence interval, we began by asking "what interval, centered at μ, has a 95% chance of including $\overline{x}$?"
- Based on the 68-95-99.7 Rule, we used interval

$$\mu \pm 2\left(\frac{\sigma}{\sqrt{n}}\right)$$

which we reversed to get the (approximate) 95% confidence interval for μ

$$\overline{x} \pm 2\left(\frac{\sigma}{\sqrt{n}}\right).$$

The Level of Confidence

- However, if we use the normal table, we find that the interval should be
$$\mu \pm 1.96 \left(\frac{\sigma}{\sqrt{n}} \right).$$
- Thus, the more precise (and correct) 95% confidence interval for the mean is
$$\overline{x} \pm 1.96 \left(\frac{\sigma}{\sqrt{n}} \right).$$

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- Find the coefficients that would give us 90% and 99% confidence intervals.
- We use the symbol z^* to represent this value.
- That allows us to write a general formula for the confidence interval for the mean.

$$\overline{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}} \right).$$

Example

Example (99% Confidence Interval)

- A diet pill is given to 100 subjects.
- After 4 weeks, the subjects have lost an average of 4.8 lbs.
- Assume a population standard deviation of $\sigma = 2.2$ lbs.
- Find a 99% confidence interval of the mean weight loss.

Outline

Improving the Margin of Error

- We have seen that we can increase or decrease the level of confidence by choosing different values of z^*.
- What effect does that have on the margin of error?
- How can we increase or decrease the margin of error without changing the level of confidence?

Improving the Margin of Error

- The margin of error is
$$z^* \left(\frac{\sigma}{\sqrt{n}} \right).$$
- The value of σ is a characteristic of the population, so we cannot change that.
- However, we can change z^* and n.

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- However, we can change z^* and n.
- How would we modify the level of confidence in order to decrease the margin of error?
- Is this a good idea?
- How else can we decrease the margin of error?

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- Is it worth it?

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- In the weight-loss example, calculate the 95% confidence interval and report on the effect on the margin of error.

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- Do the same with a 90% confidence interval.

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- Double the sample size to $n = 200$, compute the 99% confidence interval, and report on the effect on the margin of error.

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- In the weight-loss example, calculate the 95% confidence interval and report on the effect on the margin of error.
- Do the same with a 90% confidence interval.
- Double the sample size to $n = 200$, compute the 99% confidence interval, and report on the effect on the margin of error.
- Double the sample size again (to 400) and report on the effect.

Example

- In the weight-loss example, calculate the 95% confidence interval and report on the effect on the margin of error.
- Do the same with a 90% confidence interval.
- Double the sample size to $n = 200$, compute the 99% confidence interval, and report on the effect on the margin of error.
- Double the sample size again (to 400) and report on the effect.
- How large must the sample size be in order to have a 99% confidence interval AND a margin of error of 0.1 lb?

Outline

Assignment

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- Read Sections 16.3, 16.4.
- Apply Your Knowledge: 5, 6, 8, 9.
- Check Your Skills: 11, 12, 13, 15, 16.
- Exercises 19, 20, 21, 28.